MATH 3331 HOMEWORK

PROFESSOR WAGNER

(1) A spring is hung from the ceiling. When a 3-kg mass is added to the bottom end of the spring, the spring stretches 21 cm. Then the mass is pulled down another 5 cm and is released from rest.
   (a) Set up the initial value problem that models this experiment. Assume that no damping is present.

   (b) Solve the initial value problem of part 1a

(2) Use the substitution $v = y'$ to write the second order equation:

$$y'' + 4y' + 5\sin(y) = 0,$$

as a system of two first-order differential equations (planar system).

(3) Use Definition 1.22, p 141, to explain why $y_1(t) = t^3$ and $y_2(t) = t^5$ are linearly independent solutions to $t^2y''(t) - 7ty'(t) + 15y(t) = 0$.

(4) Show that $y_1(t) = t^4$ and $y_2(t) = t$ form a fundamental set of solutions for $t^2y''(t) - 4ty'(t) + 4y(t) = 0$. Then find a solution which satisfies $y(1) = 2$ and $y'(1) = -3$.

(5) Given a spring-mass system with mass $m = 1$ kg, damping $\mu = 6$ kg/s, and spring constant $k = 9$ kg/s^2, use a numerical solver (pplane.jar) to solve the initial value problem:

$$my'' + \mu y' + ky = 0, \qquad y(0) = 3, v(0) = y'(0) = -2$$

Use pplane's "keyboard input" to set the initial conditions. Then:
   (a) provide separate plots of the position versus time (y vs. t) and the velocity versus time (v vs. t),

   (b) provide a combined plot of both position and velocity versus time,

   (c) provide a plot of the velocity versus position (v vs. y) in the $y - v$ phase plane.
For each plot, choose a viewing window that highlights the important features of the solutions.

(6) Repeat problem 5 with μ changed to 1 kg/s.

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(7) Find the general solution of these differential equations:
   (a) $y'' + 2y' - 15y = 0$.

   (b) $y'' + 2y' + y = 0$.

   (c) $y'' + 2y' + 5y = 0$.

(8) Solve the initial value problem:

$$y'' + 16y = 0, \qquad y(0) = 3, \quad y'(0) = 4.$$

Find the amplitude, frequency, and phase of the solution.