

MATH 3331 HOMEWORK DUE APRIL 9

PROFESSOR WAGNER

- (1) For the initial value problem:

$$y'' + 2y' + 10y = 0, \quad y(0) = 2, \quad y'(0) = 3,$$

find an equivalent initial value problem for a first order system. Use a numerical solver to plot the solution to this initial value problem in $-5 \leq y \leq 5$, $-5 \leq y' \leq 5$. (Use pplane with “Keyboard Input of Initial Value.”)

- (2) Use a numerical solver to draw a phase plane for the “competing species” model:

$$x'(t) = (1 - x - y)x$$

$$y'(t) = (4 - 2x - 7y)y$$

in $-1 \leq F \leq 1.1$, $-1 \leq S \leq 1.1$. Draw the nullclines and indicate the direction of $(x'(t), y'(t))$ along each nullcline. Also identify and draw all of the equilibrium points. (pplane will do this for you).

- (3) Use a numerical solver to draw a phase plane for the “Predator-Prey” model:

$$F'(t) = (0.4 - 0.1S)F$$

$$S'(t) = (0.005F - 0.3)S$$

in $-2 \leq F \leq 120$, $-2 \leq S \leq 80$. Draw the nullclines and indicate the direction of $(F'(t), S'(t))$ along each nullcline. Also identify and draw all of the equilibrium points. (pplane will do this for you).

- (4) Put this system in the form $\mathbf{x}'(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{f}(t)$:

$$x_1'(t) = 3tx_1 - \sin(t)x_2 + t^2x_3 + \cos(2t)$$

$$x_2'(t) = x_1 + 5t^2x_2 + \frac{1}{t}x_3 + e^t$$

$$tx_3'(t) = tx_1 - x_3 + 2.$$

Identify any values of t_0 or \mathbf{x}_0 for which an initial condition $\mathbf{x}(t_0) = \mathbf{x}_0$ does not satisfy the requirements of the Existence and Uniqueness theorem, Theorem 3.2 on page 348.