MATH 3331 HOMEWORK DUE APRIL 18

PROFESSOR WAGNER

(1) Verify that $\{\mathbf{y}_1(t),\ \mathbf{y}_2(t)\}$ is a linearly independent set of solutions to the given system:

$$\mathbf{y}_1(t) = \begin{pmatrix} -2e^{3t} \\ e^{3t} \end{pmatrix}, \quad \mathbf{y}_2(t) = \begin{pmatrix} -e^{2t} \\ e^{2t} \end{pmatrix}, \quad \mathbf{y}'(t) = \begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix} \mathbf{y}(t)$$

Then find the solution $\mathbf{y}(t)$ to this system with $\mathbf{y}(0) = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$.

(2) Let $A = \begin{pmatrix} 3 & 1 & 0 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 3 \end{pmatrix}$.

(a) Calculate $A - 3I$.

(b) Calculate $(A - 3I)^2$.

(c) Calculate $(A - 3I)^3$.

(d) Calculate $(A - 3I)^4$.

(e) Use $A = (A - 3I) + 3I$ to calculate $e^{At} = \sum_{n=0}^{\infty} \frac{A^n t^n}{n!} = e^{3It} e^{A-3It}$. (Just calculate $e^{3It} e^{A-3It}$).

(3) Find the general solution of:

(a) $\mathbf{x}'(t) = \begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix} \mathbf{x}$ and find the solution with $\mathbf{x}(0) = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$.

(b) $\mathbf{x}'(t) = \begin{pmatrix} 5 & -1 \\ 4 & 1 \end{pmatrix} \mathbf{x}$ and find the solution with $\mathbf{x}(0) = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$.

(c) $\mathbf{x}'(t) = \begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix} \mathbf{x}$ and find the solution with $\mathbf{x}(0) = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$.

(d) $\mathbf{x}'(t) = \begin{pmatrix} 3 & -5 \\ 5 & -3 \end{pmatrix} \mathbf{x}$ and find the solution with $\mathbf{x}(0) = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$.

Date: April 12, 2018.

(4) Use pplane to draw phase portraits of the systems in #3. Have pplane sketch 4 distinct solution curves.