MATH 3334
HOMEWORK #3 DUE SEPT. 11

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(1) Give an example of a closed set $S \subset \mathbf{R}$ and a continuous function $f : \mathbf{R} \to \mathbf{R}$ such that $f(S)$ is not closed.

(2) Give an example of a bounded set $S \subset \mathbf{R} \setminus \{0\}$ and a real valued function f that is defined and continuous on $\mathbf{R} \setminus \{0\}$ such that $f(s)$ is not bounded.

(3) Show that if $f : \mathbf{R}^n \to \mathbf{R}^m$ is continuous everywhere and $S \subset \mathbf{R}^n$ is bounded, then $f(S)$ is bounded.

(4) Suppose $S \subset \mathbf{R}^n$ is compact, $f : S \to \mathbf{R}$ is continuous, and $f(\mathbf{x}) > 0$ for every $\mathbf{x} \in S$. Show that there is a number $c > 0$ such that $f(\mathbf{x}) \geq c$ for every $\mathbf{x} \in S$.

(5) (A generalization of the nested interval theorem) Suppose $\{S_k\}$ is a sequence of nonempty compact suubsets of $\mathbf{R}^n$ such that $S_1 \supset S_2 \supset \S_3 \supset \ldots$. Show that $\cap_{k=1}^{\infty} S_k \neq \phi$. (This can be done with either the Bolzano-Weierstrass theorem of the Heine-Borel theorem. Can you find both proofs?)