Find the derivative of each of the following functions:

(1) $f(x, y, z) = x^y$
(2) $f(x) = x^x$
(3) $f(x, y, z) = \sin(x \sin(y))$
(4) $f(x, y, z) = \sin(x \sin(y \sin(z)))$
(5) $f(x, y, z) = x^{(y^z)}$
(6) $f(x, y, z) = (x^y)^z$
(7) $f(x, y, z) = x^{(y+z)}$
(8) $f(x, y, z) = (x + y)^z$
(9) $f(x, y) = \sin(xy)$

Use the Fundamental Theorem of Calculus where needed to find the derivative of:

(10) $f(x, y) = \int_a^{x+y} g(t)\ dt$
(11) $f(x, y) = \int_a^{xy} g(t)\ dt$
(12) $f(x, y, z) = \int_{xy}^{\sin(x \sin(y \sin(z)))}$
(13) A function $f"\mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$ is *bilinear* if for all $x,\ y \in \mathbb{R}^n$ and all $w,\ z \in \mathbb{R}^m$, and all $a \in \mathbb{R}$:
- $f(x + ay, z) = f(x, z) + af(y, z)$
- $f(x, w + az) = f(x, w) + af(x, z)$

(a) Prove that if $f$ is bilinear, then

$$\lim_{(h,k)\to(0,0)} \frac{|f(h, k)|}{|(h, k)|} = 0. \tag{0.1}$$

(b) Prove that $Df(a, b) \cdot (h, k) = f(a, k) + f(h, y)$.