MATH 3334
HOMEWORK # 1, DUE MONDAY, AUGUST 26

PROFESSOR WAGNER

(1) Let $\mathbf{x} = (-1, -1, 1, 3)$ and $\mathbf{y} = (2, 1, 0 - 2)$. Compute $\|\mathbf{x}\|$, $\|\mathbf{y}\|$, and the angle between $\mathbf{x}$ and$\mathbf{y}$.

(2) Suppose the vectors $\mathbf{x}_j,\ j = 1, \ldots,\ k$ are mutually orthogonal, that is, $\mathbf{x}_i \cdot \mathbf{x}_j = 0$ for $i \neq j$. Show that $\|\mathbf{x}_1 + \cdots + \mathbf{x}_k\|^2 = \|\mathbf{x}_1\|^2 + \cdots + \|\mathbf{x}_k\|^2$.

(3) Show that $|\|\mathbf{x}\| - \|\mathbf{y}\|| \leq \|\mathbf{x} - \mathbf{y}\|$ for every $\mathbf{x}$, $\mathbf{y} \in \mathbb{R}^n$.

(4) For each of the following subsets S of $\mathbb{R}^2$,
   (a) Sketch S.
   (b) Determine whether S is open, closed, or neither.
   (c) Give set-descriptors for the interior of S, $\overline{S}$, and the boundary of S.

   (i) $S = \{(x, y) :\ 0 < x^2 + y^2 \leq 9\}$.
   (ii) $S = \{(x, y) :\ x^2 - x < y < 0\}$.
   (iii) $S = \{(x, y) :\ x \geq 0,\ y \geq 0,\ x + y \geq 0\}$.

(5) Prove that a real number x is a least upper bound for a subset S of $\mathbb{R}$, if an only if, for every $\epsilon > 0$, there is an upper bound y for S such that $x < y < x + \epsilon$.

(6) Prove the following statements:
   - A set S is closed if and only if S contains all of its boundary points.
   - A set S is open if and only if S contains none of its boundary points.

Date: August 22, 2019.