MATH 3334
HOMEWORK # 2, DUE WEDNESDAY, SEPTEMBER 4

PROFESSOR WAGNER

(1) Give an example of a set $S \subset \mathbb{R}$ such that the interior of S is unequal to the interior of the closure of S.
(2) Show from the definition that the following sets are disconnected:
   (a) The hyperbola $\left\{(x, y) : \ y^2 - x^2 = 1\right\}$.
   (b) Any finite set in $\mathbb{R}^n$ with at least two elements.
   (c) $\{(x, y, z) : xyz > 0\}$.
(3) Suppose that S and T are connected sets in $\mathbb{R}^n$, and that $S \cap T$ is not empty. Show that $S \cup T$ is connected.
(4) Give an example of a connected set in $\mathbb{R}^2$ for which the interior is disconnected.
(5) Suppose $\|p_{k+1} - q\| \leq c \|p_k - q\|$ for all $k \in \mathbf{Z}^+$, with $0 < c < 1$. Prove that $p_n \to q$.
(6) Prove: If $a_n \leq x_n \leq b_n$ and $\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n = L$, then $\lim_{n\to\infty} x_n = L$. I call this the "two policemen theorem".

Date: August 27, 2019.