Math 3335 Homework Solutions

p 8 #14 Derive a formula for a vector that bisects the angle between two vectors $\vec{A}$ and $\vec{B}$.

Solution The problem is easy if $|\vec{A}| = |\vec{B}|$, because then the angle bisector is $\frac{\overrightarrow{A+B}}{2}$.

(10)

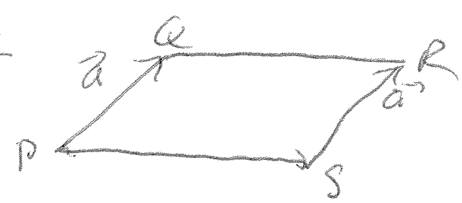
So rescale $\vec{A}$ and $\vec{B}$ to be the same length:

$$\text{angle bisector} = \frac{\vec{u}_A + \vec{u}_B}{2} = \left(\frac{A}{|A|} + \frac{B}{|B|}\right)\cdot\frac{1}{2}$$

(The 1/2 is not needed).

p 23 #7 Using vector methods, prove directly that if two sides of a quadrilateral are equal in magnitude and parallel, then the other two sides are also.

(6) Solution



$\overrightarrow{PQ} = \overrightarrow{SR} = a$

Let $\vec{b} = \overrightarrow{PS}$.

Then $\overrightarrow{QR} = \vec{a} + \vec{b} - \vec{a} = \vec{b} = \overrightarrow{PS}$. //

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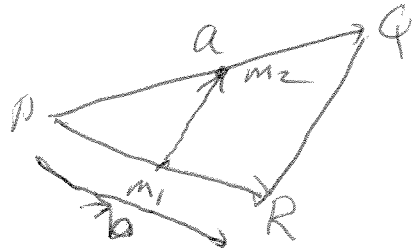
p23 #8. By vector methods, show that the line segment joining the midpoints of two sides of a triangle is parallel to the third side, and has length equal to one half the length of the third side.

Solution

8



$\vec a = \overrightarrow{PQ}$

$\vec b = \overrightarrow{PR}$.

midpoints: $P + \frac{\overrightarrow{PQ}}{2} = m_2$

$P + \frac{\overrightarrow{PR}}{2} = m_1$

$$m_1 m_2 = \frac{\vec a - \vec b}{2} = \left(P + \frac{\vec a}{2}\right) - \left(P + \frac{\vec b}{2}\right)$$

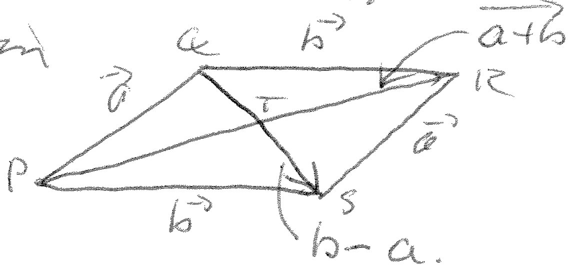
is $\parallel$ $\vec a - \vec b = \overrightarrow{RQ}$

& $\left\| \frac{a-b}{2} \right\| = \frac{1}{2}\|a-b\|$ //

p23 #9 Show that the diagonals of a parallelogram bisect each other.

Solution

8



1. Show $\underbrace{\vec a + \frac{1}{2}(\overrightarrow{b-a})}_{PT} = \frac{a+b}{2} = \underbrace{\frac{b-a}{2} + a}_{TR}$ (easy)

2. Show $\underbrace{b - \frac{a+b}{2}}_{TS} = \frac{b-a}{2} = \underbrace{\frac{a+b}{2} - a}_{QT}$. (easy)

1 Shows that $\overrightarrow{PT} = \overrightarrow{TR}$, and T = midpt $\overrightarrow{QS}$.

2 Shows that $\overrightarrow{QT} = \overrightarrow{TS}$, T = midpt $\overrightarrow{PR}$.

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(1) Suppose $\vec{X} = x_1\vec{i} + x_2\vec{j} + x_3\vec{k}$, $\|\vec{X}\| = 5$, $x_3 \geq 0$
& X has direction cosines
$\cos\alpha = 1/2 = \cos\beta$. Find X.

Solution

$(\cos\gamma)^2 = 1 - \cos^2\alpha - \cos^2\beta = 1/2.$

(6)

so $\cos\gamma = \sqrt{2}/2$ since $x_3 \geq 0$.

$(\gamma = \pi/4)$.

Then $\vec{X} = 5\cdot(\cos\alpha\vec{i} + \cos\beta\vec{j} + \cos\gamma\vec{k})$

$= 5/2\, i + 5/2\, j + \frac{5\sqrt{2}}{2}\vec{k}$

2. Consider the two lines: $\vec{r} = -2i + 3j + 7k + t(3i + 2j - k)$
$\vec{R} = -i + 5j + 8k + s(-2i + 2k)$.

a. Find the point(s) of intersection, if any.

Solution set $r = R$,

(10)

$-2i + 3j + 7k + t(3i + 2j - k) = -i + 5j + 8k + s(-2i + 2k)$.

Then

(x) $-2 + 3t = -1 - 2s$
(y) $3 + 2t = 5$
(z) $7 - t = 8 + 2s$.

$3t + 2s = 1$
$2t = 2$
$-t - 2s = 1$
Then $t = 1$, $2s = 1 - 3 = -2$
$-2s = 2$.
$s = -1$.

Check that $t=1$ $s=-1$ solves all 3 equations.

Then $r = (1, 5, 6)$
$R = (1, 5, 6)$ ✓

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2 b Assume that the lines intersect and find the cosine of the angle of intersection.

Solution line 1 has $d_1 = 3i + 2j - k$

line 2 has $d_2 = -2i + 2k$

5

$$\cos\theta = \frac{d_1 \cdot d_2}{\|d_1\| \, \|d_2\|} = \frac{-6-2}{\sqrt{9+4+1}\,\sqrt{4+4}}$$

$$= \frac{-8}{\sqrt{14}\; 2\sqrt{2}} = \frac{-2}{\sqrt{7}} = \frac{-2\sqrt{7}}{7}$$