Math 3335 Homework Solutions

p 256 #3 Sketch the region whose volume is represented by the triple integral

$$\int_0^2 \int_0^3 \int_0^{\sqrt{9-y^2}} dx\, dy\, dz$$

Solution The region is described by the inequalities

$0 \le x \le \sqrt{9-y^2} \quad 0 \le y \le 3 \quad 0 \le z \le 2$

or $0 \le x,\ 0 \le y,\ x^2+y^2 \le 9,\ 0 \le z \le 2$

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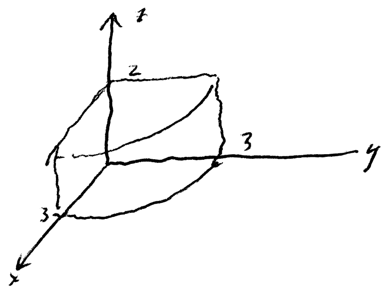

p. 262 #5 Evaluate $\iint_S F \cdot n \, dS$ when $F = y^2 x i + x^2 y j + z^2 k$

and S is the boundary of the cylinder $x^2+y^2 \le 4,\ 0 \le z \le 2$.

Solution By the divergence theorem, $\iint_S F \cdot n\, dS = \iiint_D \nabla \cdot F\, dV$

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$$= \int_0^2 \int_{-2}^2 \int_0^{\sqrt{4-y^2}} y^2 + x^2 + 2z \, dx\, dy\, dz$$

$$= \int_0^2 \int_0^{2\pi} \int_0^2 (r^2+z)\, r\, dr\, d\theta\, dz = 2\pi \int_0^2 \frac{r^4}{4} + z\frac{r^2}{2} \Big|_{r=0}^2 dz$$

$$= 2\pi \int_0^2 4 + 2z\, dz = 2\pi \left(4z + z^2\right)\Big|_{z=0}^2 = 2\pi(8+4) = 24\pi$$

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p262 #13 Evaluate $\iint_S (\nabla \times F)\cdot n\,dS$, $F = 2y\,i + (x - 2x^3z)\,j + xy^3\,k$

$S: x^2+y^2+z^2=1,\ z \geq 0.$ (Assume n points up)

Solution By Stokes' theorem, $\iint_S (\nabla\times F)\cdot n\,dS = \oint_C F\cdot dR$

where C is the curve $R(\theta) = \cos\theta\, i + \sin\theta\, j$ $0 \leq \theta \leq 2\pi$

(14) Then on C $x = \cos\theta$, $y = \sin\theta$, $z = 0$

$R'(\theta) = -\sin\theta\, i + \cos\theta\, j$

$$\oint_C F\cdot dR = \int_0^{2\pi} (2\sin\theta\, i + \cos\theta\, j + \cos\theta\sin^3\theta\, k)\cdot(-\sin\theta\, i + \cos\theta\, j)\,d\theta$$

$$= \int_0^{2\pi} -2\sin^2\theta + \cos^2\theta\,d\theta = \int_0^{2\pi} \cos(2\theta) - \sin^2\theta\,d\theta$$

$$= 0 - \int_0^{2\pi} \frac{1-\cos 2\theta}{2}\,d\theta = -\pi$$

Alternate Solution By the divergence theorem,

$$\iint_S (\nabla\times F)\cdot n\,dS + \iint_D (\nabla\times F)\cdot(-k)\,dx\,dy = \iiint_{\substack{z\geq 0\\ x^2+y^2+z^2\leq 1}} \mathrm{div}(\nabla\times F)\,dV = \iiint 0\,dV$$

by the divergence theorem and $\nabla\cdot(\nabla\times F) = 0$. $D = \{z=0,\ x^2+y^2\leq 1\}$

So $$\iint_S (\nabla\times F)\cdot n\,dS = \iint_D \nabla\times F\cdot k\,dS = \iint_D -1\,dx\,dy = -\pi$$

$-1 = (\nabla\times F)\cdot k$ on $z=0$.