Math 3335 Homework Solutions

p 60 #1 Derive the identity
$$(A\times B)\times(C\times D) = [A,B,D]C - [A,B,C]D.$$

Solution Use (1.30) $A\times(B\times C) = (A\cdot C)B - (A\cdot B)C$

Replace A with $A\times B$, B with C, C with D

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$$(A\times B)\times(C\times D) = ((A\times B)\cdot C)D - ((A\times B)\cdot D)C$$
$$= [A,B,C]D - [A,B,D]C. \quad //$$

#3. Derive the identity $A\times(B\times C) + B\times(C\times A) + C\times(A\times B) = 0.$

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Solution. Use 1.30 on each term
$$= (A\cdot C)B - (A\cdot B)C + (B\cdot A)C - (B\cdot C)A + (C\cdot B)A - (C\cdot A)B$$
$$= 0.$$

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p60 # 8. Let $A = 3i + j + 2k$
$B = 4i + j + 5k$
$C = i - j + k.$

3 Find $A \times B$: $= \begin{vmatrix} i & j & k \\ 3 & 1 & 2 \\ 4 & 1 & 5 \end{vmatrix} = i(5-2) - j(15-8) + k(3-4)$

$= 3i - 7j - k.$

3 Find $[A, B, C]$: $= \begin{vmatrix} 3 & 1 & 2 \\ 4 & 1 & 5 \\ 1 & -1 & 1 \end{vmatrix} = 3\cdot(1+5) - 1(4-5) + 2(-4-1)$
$= 18 + 1 - 10 = 9.$

Or $= \underbrace{(3i - 7j - k)}_{A \times B} \cdot (i - j + k) = 3 + 7 - 1 = 9.$

2 Find $\| A \times B \|$: $= \| 3i - 7j - k \| = \sqrt{9 + 49 + 1} = \sqrt{59}$

2 Find the distance from the tip of C to the plane through the origin spanned by A and B.:

$= \dfrac{| C \cdot (A \times B) |}{\| A \times B \|} = \dfrac{9}{\sqrt{59}}$

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p. 60 #11 Simplify $[A\times(A\times B)]\times A\cdot C$

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I Method: Use 1.30 on $A\times(A\times B)=(A\cdot B)A-(A\cdot A)B$

Then

$$([A\times(A\times B)]\times A)\cdot C=([(A\cdot B)A-|A|^2B]\times A)\cdot C$$

$$=-|A|^2(B\times A)\cdot C=|A|^2[A,B,C].$$

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p70 #3 Find $f'(t)$:

a. $f(t) = (3t\,i + 5t^2 j)\cdot(t\,i - \sin t\, j)$

$= 3t^2 - 5t^2 \sin t$

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So $f'(t) = 6t - 10t \sin t - 5t^2 \cos t$

or $f'(t) = (3i + 10t\,j)\cdot(t\,i - \sin t\, j) + (3t\,i + 5t^2 j)\cdot(i - \cos t\, j)$

$= 3t - 10 \sin t + 3t - 5t^2 \cos t$

$= 6t - 10 \sin t - 5t^2 \cos t$

b. $f(t) = |2t\,i + 2t\,j - k| = \sqrt{4t^2 + 4t^2 + 1}$

$f'(t) = \frac{1}{2} \frac{16t}{\sqrt{8t^2+1}} = \frac{8t}{\sqrt{8t^2+1}}$

6

or $f(t) = \left((2t\,i + 2t\,j - k)\cdot(2t\,i + 2t\,j - k)\right)^{1/2}$

$f'(t) = \frac{1}{2}\left[(2t\,i + 2t\,j - k)\cdot(2t\,i + 2t\,j - k)\right]^{-1/2} \cdot 2(2t\,i + 2t\,j - k)\cdot(2i + 2j)$

$= \frac{8t}{\sqrt{8t^2+1}}$

c. $f(t) = \left[(i + j - 2k) \times (3t^4 i + t\,j)\right]\cdot k$

only one factor depends on t!

6

$f'(t) = \left[(i + j - 2k) \times (12t^3 i + j)\right]\cdot k$ (product rule!)

$= \begin{vmatrix} 1 & 1 & -2 \\ 12t^3 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1 - 12t^3.$

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p70 #21. Show that $\frac{d}{dt}\left(R \times \frac{dR}{dt}\right) = R \times \frac{d^2R}{dt^2}$

6 proof $= \underbrace{\frac{dR}{dt} \times \frac{dR}{dt}}_{0} + R \times \frac{d^2R}{dt^2} = R \times \frac{d^2R}{dt^2}$

p85 #5 a Determine the arc length of the curve
$x = e^t \cos t \quad y = e^t \sin t \quad z = 0$
between $t=0$ and $t=1$.

Solution $= \int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

6 $= \int_0^1 e^t \sqrt{(\cos t - \sin t)^2 + (\sin t + \cos t)^2}\, dt$

$= \int_0^1 e^t \sqrt{\cos^2 t - 2\cos t \sin t + \sin^2 t + \sin^2 t + 2\sin t \cos t + \cos^2 t}\, dt$

$= \int_0^1 e^t \sqrt{2}\, dt = \sqrt{2}\, e^t \Big|_0^1 = \sqrt{2}(e^1 - 1).$

b Reparameterize the curve in terms of arclength

8 Solution $\frac{ds}{dt} = \sqrt{2} e^t \quad s = \int_0^t \sqrt{2} e^\tau d\tau$
$= \sqrt{2}(e^t - 1)$

$\sqrt{2} e^t = s + \sqrt{2}$
$e^t = \frac{1}{\sqrt{2}}(s + \sqrt{2}) \quad t = \ln\left(\frac{(s+\sqrt{2})}{\sqrt{2}}\right)$

We could also use $t = \ln(s/\sqrt{2}) \quad (s = \sqrt{2} e^t)$

Then $x = \frac{s}{\sqrt{2}}\left(\cos(\ln(s/\sqrt{2}))\right) \quad y = s/\sqrt{2}\, \sin(\ln(s/\sqrt{2})).$

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p 85 # 5c This curve is a spiral. Sketch it to see why

Sketch:

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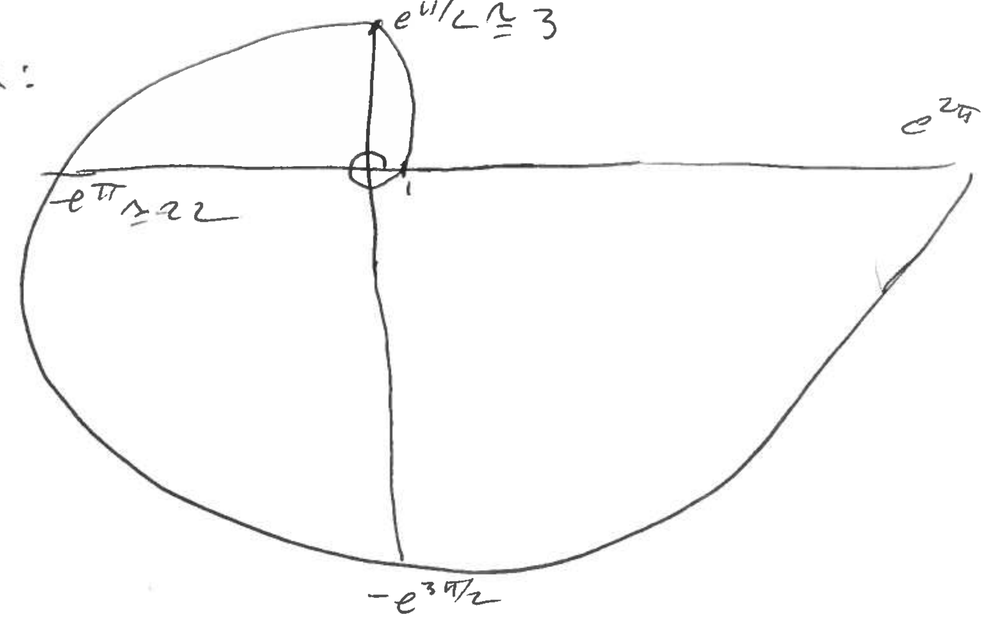

p 85 # 8. Show that the curve $x=t$, $y=2t^2$, $z=t^3$ intersects the plane $x+8y+12z=162$ at right angles.

(6)

Solution first, find t where the curve intersects the plane

$$\underbrace{t}_{x} + 8\underbrace{(2t^2)}_{y} + 12\underbrace{(t^3)}_{z} = 162$$

This is almost impossible to solve, but we guess $t=2$ and it works!

$r'(t) = i + 4t\,j + 3t^2 k = i + 8j + 12k$ at $t=2$.

This is the normal vector to the plane.

So the curve intersects the plane at right angles

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p85 #12 Parameterize a right-handed helix with unit pitch that is wrapped around the cylinder described by $y^2+z^2=1$.

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Solution: (j,k,i) form a right handed triple. So a right handed helix with unit pitch on $y^2+z^2=1$ is

$$R(t) = \cos t\, j + \sin t\, k + \frac{t}{2\pi} i$$