MATH 3338 **FINAL EXAM** **NAME________________**

May 8, 2000 **ID #________________**

1. 10 entrants in a baking contest bake one pie each. There are 6 pecan pies and 4 key-lime pies. Each pie has its baker's name labeled on the bottom.
   a. In how many ways can the 10 pies be placed in a single line? 6 pts

   b. If the six pecan pies are put on one table, and the 4 key-lime pies are put on another table, how many ways are there to line up the two groups of pies on their respective tables? 6 pts

   c. The judges assign a grade of "outstanding," "excellent," or "good enough for the UH cafeteria," to each of the 10 pies. How many different scorecards are possible? 8 pts

   d. Five pies are chosen at random. What is the probability that exactly 3 of the pecan pies are among the chosen? 10 pts

2. Let X be the weight, in ounces, of an orange selected at random from a certain crop. Oranges weighing less than 3 ounces are regarded as defective. Suppose X is normal with mean 4 and variance .25 (N(4,.25)).
   a. What is $P(X \leq 3)$? 10 pts

   b. In a bag of 12 oranges from this crop, what is the probability that 2 weigh less than 3 ounces? 10 pts

3. Let T be an exponential random variable with mean 5.
   a. Find $P(T > 5)$. 10 pts

   b. Find $P(T > 8 \mid T > 3)$. 10 pts

4. Suppose X is a random variable with moment generating function

   $$M(t) = \frac{1}{3}e^{-3t} + \frac{1}{6} + \frac{1}{4}e^{2t} + \frac{1}{4}e^{4t}$$

   a. Find E(X). 10 pts

   b. Find Var(X). 10 pts

c. Find P(X < 0). 12 pts

5. You have a random number generator for the uniform distribution on the interval [0,1]. If Z is a random variable representing this random number generator, what function of Z, X = g(Z), will have the c.d.f. $F(x) = \sqrt{x/4},\ 0 \le x \le 4$?

6. You are rolling two ordinary six-sided dice.
   a. What is the probability that the first "doubles" (i.e., both dice match) occurs on or after the 4^{th} roll?

   b. What is the probability that the third "doubles" occurs on the 20^{th} roll? 12]

7. You are responsible for watching a cloud chamber to count "cosmic ray" particles. These particles enter the cloud chamber at a rate of 1 every 20 minutes. You may stop watching after you have counted another 3 comic ray particles. You are supposed to meet a friend for lunch in 60 minutes. How do you calculate the probability that you will be finished watching the cloud chamber in 60 minutes? 12 pts

8. X and Y are independent random variables. X and Y have the same p.d.f:

$$f(x) = \begin{cases} 1/6, & x = 0 \\ 1/3, & x = 1 \\ 1/2, & x = 2 \end{cases}$$

Find the p.d.f for Z = X + Y. 14]

9. A random rectangle has sides of length X and Y, where X and Y are independent random variables with pdfs:

$$X: f(x) = \frac{1}{3}e^{-x/3}, \quad x \ge 0,$$

$$Y: g(y) = \frac{1}{4}xe^{-x/2}, \quad x \ge 0.$$

What is the expected value for the area of this rectangle? 10 pts

10. Let X be a random variable with density function $f(x) = \frac{1}{2\sqrt{x}}, \quad 0 < x \le 1$

Find E(X) and Var(X). 12 pts

11. A bag of tulip bulbs contains 50 bulbs for pink tulips, 20 bulbs for purple tulips, and 5 bulbs for black tulips. In your experience, the germination rates for pink, purple, and black tulip bulbs are 50%, 60% and 30%, respectively. You pick one bulb from the bag, and plant it. It germinates. What is the probability that it will produce black flowers?

14 pts