Math 3338 Homework Solutions

Ch 3 #4 2 dice rolled, $P\{\text{at least one } 6, \mid \text{sum}=i\}$ $i=2,3,\dots,12$.

Solution If one die is 6, the sum ≥ 7.

Or, if sum <7, both dice <6.

So $P\{\text{at least one } 6 \mid \text{sum}=i\} = 0$ if $i=2,3,4,5,6$.

There are 6 outcomes with sum $=7$, 2 have one 6 $\rightarrow$ $(6,1),(1,6)$

So $P\{\text{at least one } 6 \mid \text{sum}=7\} = 2/6 = 1/3$

There are 5 outcomes with sum $= 8$; 2 have one 6 $(6,2),(2,6)$.

So $P\{\text{at least one } 6 \mid \text{sum}=8\} = 2/5$.

Similarly $P\{\text{at least one } 6 \mid \text{sum}=9\} = 2/4 = 1/2$.

$P\{\text{at least one } 6 \mid \text{sum}=10\} = 2/3$

$P\{\text{at least one } 6 \mid \text{sum}=11\} = 1$

$P\{\text{at least one } 6 \mid \text{sum}=12\} = 1$.

Or $P\{\text{at least one } 6 \mid \text{sum}=i\} = \begin{cases} 0 & i=1,2,3,4,5,6 \\ \dfrac{2}{13-i} & i=7,\to 11 \\ 1 & i=12. \end{cases}$

#62 $P(\text{Barbara's shot hits}) = p_1$ $\qquad$ $B =$ "Barbara's shot hits"

$P\{\text{Dianne's shot hits}\} = p_2$ $\qquad$ $D =$ "Dianne's shot hits."

a, $P(B\cap D \mid B\cup D) = \dfrac{P(B\cap D)}{P(D)} = \dfrac{p_1 p_2}{p_1+p_2-p_1p_2}$

We used independence of B and D, and Prop. 4.3 p. 28.

b. $P(B \mid B\cup D) = \dfrac{P(B)}{P(B\cup D)} = \dfrac{p_1}{p_1+p_2-p_1p_2}$.

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Ch 3 # 66 a. For current to flow between A and B,

either #1 and #2 must function correctly, or

#3 and #4 " " " .

A_i = "switch #i functions correctly"

$P(\text{current flows}) = P((A_1 \cap A_2) \cup (A_3 \cap A_4))$

$P(A_1 \cap A_2) = p_1 p_2$ because relays #1 & #2 function independently.

$P(A_3 \cap A_4) = p_3 p_4$ " " 3 & 4 " "

So $P(\text{current flows}) = p_1 p_2 + p_3 p_4 - P((A_1 \cap A_2 \cap A_3 \cap A_4))$

$= p_1 p_2 + p_3 p_4 - p_1 p_2 p_3 p_4$ by independence of A_1, A_2, A_3, A_4.

b. Now for current to flow, we need

$A_1 \cap A_4$ or $A_2 \cap A_5$, if A_3^c has occurred

or, if A_3 occurs, we need $(A_1 \cup A_2) \cap (A_4 \cup A_5)$

So $P(\text{current flows}) = P((A_1 \cap A_4) \cup (A_2 \cap A_5) \mid A_3^c)\, P(A_3^c)$

$+ P((A_1 \cup A_2) \cap (A_4 \cup A_5) \mid A_3)\, P(A_3)$

Since $A_1, \ldots, A_5$ are indep.,

this $= ((p_1 p_4) + (p_2 p_5) - p_1 p_4 p_2 p_5)(1 - p_3) + ((p_1 + p_2 - p_1 p_2)(p_4 + p_5 - p_4 p_5)) p_3$

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Ch 3 # 76 $E \cap F = \emptyset$ Exper. repeated w/ indep trials.

$P(E \text{ before } F) = ?$

Solution $= P(E \text{ on } 1^{st} \text{ try}) + P(E \text{ on } 2^{nd} \text{ try} \mid E^c \cap F^c \text{ on } 1^{st} \text{ try}) P(E^c \cap F^c)$

$+ P(E \text{ on } 3^{rd} \text{ try} \mid E^c \cap F^c \text{ on } 1^{st} + 2^{nd} \text{ trial}) P(E^c \cap F^c)^2 + \cdots$

trials indep $\Rightarrow$

$= P(E) + P(E) \cdot P(E^c \cap F^c) + P(E) P(E^c \cap F^c)^2 + \cdots + P(E) (P(E^c \cap F^c))^n + \cdots$

$P(E^c \cap F^c) = P(\overline{E \cup F}) = 1 - P(E \cup F) = 1 - P(E) - P(F)$

So $P(E \text{ before } F)$

$= P(E) \sum_{n=0}^{\infty} P(E^c \cap F^c)^n = \dfrac{P(E)}{1 - P(E^c \cap F^c)}$

$= \dfrac{P(E)}{1 - (1 - P(E) - P(F))}$

$= \dfrac{P(E)}{P(E) + P(F)}$

#77 Unending seq. of indep. trials, each equally likely to have outcome 1, 2, or 3.

a. Find $P(1^{st}$ trial results in $1 \mid 3$ is last$)$

Solution If 3 occurs last, it cannot occur on 1^{st} trial

So $P(1^{st} \text{ is } 1 \mid 3 \text{ is last})$

$= 1/2$.

b Find $P(1^{st}\ 2$ trials result in $1 \mid 3$ is last$)$

Solution If 3 is last, it cannot occur on 1^{st} 2 trials.

So $P(1^{st}\ 2 \text{ trials result in } 1 \mid 3 \text{ is last})$

$= (1/2)(1/2) = 1/4$ since trials are independent