Math 3338 Homework Solutions

p 163 #7 die rolled twice what are possible values for

a. Maximum of 2 rolls

(2) Solution: 1, 2, 3, 4, 5, 6

b. Minimum of 2 rolls

(2) Solution 1, 2, 3, 4, 5, 6

c. Sum of 2 roll

(2) Solution: 2, 3, ... 12

d. First roll − second roll?

(2) Solution −5, −4, −3, −2, −1, 0, 1, 2, 3, 4, 5

#8 If the die in #7 is fair, calculate probabilities assoc. with random variable in 7a–d)

a. $P(X=1) = 1/36$ $P(X=2) = 3/36$ (2,1), (1,2), (2,2)

(4) $P(X=3) = 5/36$ (3,1), (3,2), (3,3), (2,3), (1,3)

$P(X=4) = \frac{7}{36}$ (4,1), (4,2), (4,3), (4,4), (3,4), (2,4), (1,4)

$P(X=5) = 9/36$, $P(X=6) = 11/36$.

b. $P(X=1) = 11/36$ (6,1) (5,1) (4,1), (3,1), (2,1), (1,1), (1,2), (1,3), (1,4), (1,5) (1,6)

(4) $P(X=2) = 9/36 = 1/4$ $P(X=3) = \frac{7}{36}$ $P(X=4) = 5/36$ $P(X=5) = 3/36 = \frac{1}{12}$, $P(X=6) = 1/36$.

c. $P(X=2) = 1/36$ $P(X=3) = 2/36$, $P(X=4) = 3/36$, $P(X=5) = \frac{4}{36} = \frac{1}{9}$, $P(X=6) = 5/36$ $P(X=7) = 1/6$

(8) $P(X=8) = 5/36$, $P(X=9) = \frac{4}{36} = \frac{1}{9}$, $P(X=10) = 3/36 = 1/12$, $P(X=11) = 2/36 = \frac{1}{18}$

$P(X=12) = 1/36$

d. $P(X=-5) = P(X=5) = 1/36$

(4) $P(X=-4) = P(X=4) = 2/36$

$P(X=-3) = P(X=3) = 3/36$

$P(X=-2) = P(X=2) = 4/36$

$P(X=-1) = P(X=1) = 5/36$

$P(X=0) = 6/36$

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p163#9 Repeat Example 1c when the balls are selected with replacement.

Urn contains 20 balls numbered 1 to 20.

4 balls chosen with replacement. X = largest number chosen.

X could be $1, 2, 3, \dots, 20$

$P(X=1) = \frac{1}{20^4}$ There is only one way to get $X=1$

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$\approx 6.25 \times 10^{-6}$ There are 20^4 possible choices.

$P(X=2) = P(X \le 2) - P(X=1)$

$= \frac{2^4}{20^4} - \frac{1^4}{20^4} = 9.375 \times 10^{-5}$

$P(X=3) = P(X \le 3) - P(X \le 2) = \frac{3^4 - 2^4 + 1}{20^4} = .0004125$

$P(X=4) = P(X \le 4) - P(X \le 3) = \frac{4^4 - 3^4 + 2^4 - 1}{20^4} = .0011875$

$P(X=5) = P(X \le 5) - P(X \le 4) = \frac{5^4 - 190}{20^4} = \frac{435}{20^4} = .002719$

$P(X=6) = P(X \le 6) - P(X \le 5)$

$= \frac{6^4 - 435}{20^4} = \frac{1296 - 435}{20^4} = .005381$

$P(X=7) = \frac{7^4 - 861}{20^4} = \frac{2401 - 861}{20^4} = .009625$

$$P(X=n) = \frac{n^4 - \sum_{j=1}^{(n-1)} j^4 (-1)^{n-1+j}}{20^4} \quad n = 1, 2, 3, \dots, 20$$

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p163 #23 Have \$1000. Commodity sells for \$2 per ounce

In one week price will be either \$1 or \$4 per ounce

— equally likely

a To maximize expected amount of money after 1 week, what strategy?

Solution Assume that we will buy x ounces now and sell in 1 week.

(10) The cost of purchase is $\$2x$

With probability 1/2, the commodity is sold for \$1/oz, yielding $\$2x$

With prob. 1/2 the sale price is \$4, → yielding $\$4 \cdot 2x = \$8x$.

Thus the expected amount of money after 1 week

is $1000 - 2x + 1/2(2x + 8x) = 1000 + 3x$

This is maximized by making x as large as possible, $x = 500$

b. To maximize amount of commodity owned after 1 week?

Solution Assume that we invest $\$x$ now and $\$1000 - x$ in 1 week.

(10)

With prob. 1/2, the amount owned after 1 week is

$x \cdot 1/2 + (1000-x) \cdot 1$

With prob 1/2, the amount owned after 1 week is

$x \cdot 1/2 + (1000-x) \cdot 1/4$

So the expected amount owned is

$\frac{1}{2}\left(x \cdot 1/2 + (1000-x)\right) + 1/2\left(x \cdot 1/2 + (1000-x) \cdot 1/4\right)$

$= x/2 - x/2 - x/8 + 1000 = 1000 - x/8$

This is maximized by making x as small as possible, $x = 0$

So we plan to use all \$1000 in one week.

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p163 #36 Coin flipped until 1st tails, N = # flips, player wins 2^N dollars

(c) X = winnings, show $E(X) = \infty$. $X = 2^N$

Solution $E(X) = \sum_{n=1}^{\infty} 2^n p(N=n)$

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N has a geometric distribution, $p(N=n) = (1/2)^{n-1} \cdot 1/2 = (1/2)^n$

So $E(X) = \sum_{n=1}^{\infty} 2^n \cdot (1/2)^n = \sum_{n=1}^{\infty} 1 = \infty$.

a. Would you pay \$1,000,000 to play this game once?

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No, the probability of winning more than \$1,000,000

is $\sum_{n=20}^{\infty} (1/2)^n = \frac{1}{2^{20}} \cdot \frac{1}{1-1/2} = 2^{-19} = 1.9 \times 10^{-6}$

$(\log_2 (1000,000) = 19.93)$.

b. Would you be willing to pay \$1,000,000 for each game

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if you could play as long as you liked and only had to settle up when you quit playing?

Solution Yes!