Math 3338 Homework Solutions

p. 163 #43 Transmit 5 bits: 11111 or 00000 to represent 1 or 0.

Each bit transmits correctly with prob. .8

Receiver uses "majority" decoding.

What is probability of incorrect reception?

Solution Let X = # correct bits. Message is received incorrectly if

$X \le 2.$ $P(X\le 2) = P(X=0) + P(X=1) + P(X=2)$

$= (.8)^0(.2)^5 + \binom{5}{1}(.8)^1(.2)^4 + \binom{5}{2}(.8)^2(.2)^3$

(10) $= .00032 + 5(.00128) + 10(.00512)$

$\approx .05792$

Or, = binom.dist(2, 5, 0.8, 1) ↳ use cdf.

This assumes that the transmission of different bits is independent.

#54 avg # cars abandoned on a certain highway is 2.2.

a. Approximate P(no cars abandoned in next week)

(8) This is a Poisson process with $\lambda = 2.2$

Prob(no cars abandoned) $= e^{-2.2}\frac{(2.2)^0}{1} = .1108$

b. P(at least 2 cars abandoned in next week)

(8) $\approx 1 - P(\text{\# cars abandoned} \le 1) = 1 - e^{-2.2}(1+2.2) = 1 - .3546$

$= .6454$

64 Suicide rate is 1 per 100,000 per month in one state

a. City A has pop. of 400,000, X = # suicides/month in City A

$P(X\ge 8) = 1 - P(X\le 7)$

(10) X is Poisson with $\lambda = 4$

$P(X\le 7) = e^{-4}\left(1 + 4 + \frac{4^2}{2} + \frac{4^3}{6} + \frac{4^4}{24} + \frac{4^5}{120} + \frac{4^6}{720} + \frac{4^7}{5040}\right) = .94887$

$P(X\ge 8) = 1 - .94887 = .05113$ (or use poisson.dist)

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p163 #64b $P(X \geq 8$ on at least 2 months in a year)

Solution $Y =$ # months in a year on which $X \geq 8$.

(8) Y is binomial with $n = 12$, $p = .05113$

$$P(Y \geq 2) = 1 - P(Y \leq 1) = 1 - \left[(1-p)^{12} + \binom{12}{1} p (1-p)^{11}\right]$$
$$= 1 - .87714$$
$$= .12286$$

Or $= 1 -$ binom.dist(1, 12, .05113, 1)

c. $P(1^{st}$ month $X \geq 8$ is $\#i)$ $i = 1, 2, -$

(8) $$= (1-p)^{i-1} p$$
$$= (.94887)^{i-1} (.05113) \quad \text{(geometric)}$$

#72 "World Series" Best of 7 game series. Team A wins each game independently with Prob. 0.6

$P($team A wins in i games$)$ $i = 4, 5, 6, 7$?

$i = 4 \quad = \binom{4}{0}(.6)^4(.4)^0 = .1296$

$i = 5 \quad = \binom{4}{1}(.6)^4(.4) = .20736$ Negative Binomial

(16) $i = 6 \quad = \binom{5}{2}(.6)^4(.4)^2 = .2076$

$i = 7 \quad = \binom{6}{3}(.6)^4(.4)^3 = .16588$

$P($ team A wins series$) = .710208$ (= sum)

For a 3-game series

$$P(A \text{ wins series}) = P(A \text{ wins game } 1+2) + P(A \text{ wins game } 1+3 \text{ or } 2+3)$$
$$= (.6)^2 + (.6)^2(.4)\cdot\binom{2}{1}$$
$$= .36(1 + .8) = .648$$

This is less than Team A's probability of winning 4 out of 7.

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p165 #79. Batch of 100 items contains 6 defective, 94 ok.

10 items sampled. X = # defective in sample

(6) a. $P(X=0) = \frac{\binom{6}{0}\binom{94}{10}}{\binom{100}{10}} = \frac{94!\,90!}{100!\,84!}$ = hypergeom.dist(0, 10, 6, 100, 0)

$= 0.5223$

(6) b. $P(X>2) = 1 - P(X\le 2)$ = 1 - hypergeom.dist(2, 10, 6, 100, 1)

use cdf

$= 1 - .98745$

$= .01255$