Math 3338 Homework Solutions

Ch. 5 p212 #15. X is normal with $\mu = 10$, $\sigma^2 = 36$ ($\sigma = 6$)

a. $P(X>5) = P\left(\frac{X-10}{6} > \frac{5-10}{6}\right)$ $\quad Z = \frac{X-10}{6}$ is normal with $\mu=0, \sigma=1$.

$= P(Z > -5/6) = P(Z < 5/6) \doteq \Phi(5/6) = .7967$

6

Excel: 1-Norm.dist(5,10,6,1) = .7977

b. $P(4<X<16) = P\left(\frac{4-10}{6} < \frac{X-10}{6} < \frac{16-10}{6}\right) = P(-1<Z<1)$

6

$= \Phi(1) - \Phi(-1) = \Phi(1) - (1-\Phi(1)) = 2\Phi(1) - 1$

$\Phi(1) = .8413 \quad 2 \times .8413 - 1 = .6826$

6 c. $P(X<8) = P\left(\frac{X-10}{6} < \frac{8-10}{6}\right) = P(Z < .3333\ldots) \doteq .6293$

6 d. $P(X<20) = P\left(\frac{X-10}{6} < \frac{20-10}{6}\right) = P(Z < 5/3 = 1.667) \doteq .9525$

6 e. $P(X>16) = P\left(\frac{X-10}{6} > \frac{16-10}{6} = 1\right) = 1 - \Phi(1) = 1 - .8413 = .1587$

31a Fire station to be located along road of length $A < \infty$.

Fires occur uniformly on $(0, A)$, which location for the fire station minimizes the expected distance to the fire?

Minimize $E(|X-a|)$, X is uniform on $(0,A)$

Solution X has pdf $f(x) = \begin{cases} \frac{1}{A} & 0 < x < a \\ 0 & \text{otherwise} \end{cases}$

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$$E(|X-a|) = \int_0^a (a-x)\frac{1}{A}\,dx + \int_a^A (x-a)\frac{1}{A}\,dx$$

$$= \left(ax - \frac{x^2}{2}\right)\frac{1}{A}\Big|_{x=0}^{a} + \frac{1}{A}\left(\frac{x^2}{2} - ax\right)\Big|_{x=a}^{A} = \frac{a^2}{2}\frac{1}{A} + \frac{1}{A}\left(\frac{A^2}{2} - aA - \frac{a^2}{2} + a^2\right)$$

$$= \frac{1}{A}\left(a^2 + \frac{A^2}{2} - aA\right) = f(a)$$

$f'(a) = \frac{1}{A}(2a - A)$ $\begin{cases} < 0 & \text{if } a < A/2 \\ = 0 & a = A/2 \\ > 0 & a > A/2 \end{cases}$ so $a = A/2$ minimizes

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Ch 5 p 212 #38 Y is uniformly distributed over $(0,5)$

$P(4x^2+4xY+Y+2$ has real roots$)$?

Solution as a polynomial in x, with y given

$4x^2+4xy+y+2$ has discriminant B^2-4AC

(10) $= 16y^2-4\cdot4\cdot(y+2) = 16(y^2-y-2) = 16(y-2)(y+1)$

thus the discriminant is positive if $y>2$ or $y<-1$

Since Y is uniform on $(0,5)$ $P(Y<-1)=0$

and $P(Y>2) = \underline{\underline{3/5}}$ = prob of real roots

40. If X is uniformly distributed over $(0,1)$, find the density function for $Y=e^X$.

(10) Solution The cdf for Y is

$$F_Y(y) = P(e^X \le y) = P(X \le \ln y) = \begin{cases} 0 & \ln y < 0 \\ \ln y & 0<\ln y<1 \\ 1 & 1<\ln y \end{cases}$$

then Y has pdf

$$f_Y(y) = F_Y'(y) = \begin{cases} 0 & 0<y<1 \ (y<1) \\ 1/y & 1<y<e \\ 0 & e<y \end{cases}$$