Math 3338 Homework Solutions

Ch 5, p 212

#11 A point is chosen at random, on a line segment of length L. Interpret this, find the probability that the ratio of the shorter to the longer segment is < 1/4

Solution The choice of a point can be interpreted as a random variable X with a uniform distribution on $[0, L]$.

Let R be the ratio of the shorter to the longer segment

(8)

$R = \min\left(\frac{X}{L-X}, \frac{L-X}{X}\right)$

$R = \frac{X}{L-X}$ if $X < L/2$. $R = \frac{L-X}{X}$, if $X > \frac{L}{2}$.

$R < 1/4$ if $X < \frac{L}{5}$ or $X > \frac{4L}{5}$

So $P(R < 1/4) = \frac{L}{5L} + \left(L - \frac{4L}{5}\right)\frac{1}{L} = 2/5$.

#23. 1000 indep rolls of fair die. Approx prob that 6 appears between 150 + 200 times inclusively

Solution $N = \#6$'s in 1000 rolls. N has binomial dist with $n = 1{,}000$, $p = \frac{1}{6}$

Probabilities for N can be approximated by a normal dist with mean $\frac{1000}{6} = 166.\underline{6}$, Standard deviation $\sigma = \sqrt{\frac{1000}{6} \cdot \frac{5}{6}}$

(12)

$= 11.785$

With a correction for continuity, $P(150 \leq N \leq 200)$

$\doteq P(149.5 \leq X \leq 200.5)$ where X is normal with mean 166.7 and $\sigma = 11.785$

Thus $P = P\left(\frac{149.5 - 166.7}{11.785} \leq Z \leq \frac{200.5 - 166.7}{11.785}\right)$

$\simeq P(-1.46 \leq Z \leq 2.87) = .9979 - (1 - \Phi(1.46))$

$= .9979 + .9279 - 1 = 0.9258$

Math 3338 Homework Solutions

Ch 5 p 212, 23b If 6 appears exactly 200 times, find P(5 appears < 150 times)

Solution $N = \#6$'s, $M = \#5$'s. both have binomial dist with $n=1000$, $p=\frac{1}{6}$

$$P(M<150 \mid N=200) = \frac{P(M<150 \text{ and } N=200)}{P(N=200)}$$

With a correction for continuity, $P(N=200) = P\left(\frac{199.5-166.7}{11.785} < Z < \frac{200.5-166.7}{11.785}\right)$

(12) $= P(2.78 < Z < 2.87) = .9979 - .9973 = .0006$

For $P(M<150$ and $N=200)$. there are $1000-200=800$ rolls that might be 5.

Under this condition M is binomial with $n=800$, $p=1/6$

Approx this prob. with a normal dist with mean $800 \cdot \frac{1}{6} = 133.3$

and $\sigma = \sqrt{800 \cdot \frac{1}{6} \cdot \frac{5}{6}} = 10.541$

Then with correction for continuity

$P(M<150$ and $N=200) = P\left(\frac{M-133.3}{10.541} < \frac{149.5-133.3}{10.541}\right)$

$= P(Z < 1.54) = .9382$

27. In 10,000 independent tosses of a coin, 5800 heads.

Is it reasonable to conclude the coin is not fair?

Solution. If the coin is fair and $N = \#$ Heads

$P(N \geq 5800) \cong P\left(\frac{N-5000}{50} \geq \frac{5800.5-5000}{50}\right) = P(Z \geq \frac{800.5}{50} = 16.1)$

(10) $50 =$ std dev of $N = \sqrt{10000 \cdot \frac{1}{2} \cdot \frac{1}{2}}$

This prob is essentially 0.

Since we observed an outcome that is nearly impossible under the assumption that the coin is fair, it must be that the coin is not fair.

Math 3338 Homework Solutions

Ch. 5 p 212

33. The lifetime of a radio, in years, is exponentially distributed with $\lambda = 1/8$.

Jones buys a used (working) radio.

What is P(it works for 8 or more additional years)?

Solution Since the exponential dist is memoryless,

(6) the desired conditional prob $P(X > 8+a \mid X > a)$ (a = age at purchase)

$= P(X>8) = e^{-8/8} = e^{-1} = .3679$

35. The lung cancer hazard rate $\lambda(t)$ for a t-year-old male smoker is

$$\lambda(t) = .027 + .00025(t-40)^2 \quad t \geq 40$$

Assuming that a 40-year-old male smoker survives all other hazards, what is the probability that he survives to

(a) age 50 and (b) age 60 without contracting lung cancer?

a. $= \exp\left(-\int_{40}^{50} \lambda(t)\,dt\right)$

$\int_{40}^{50} \lambda(t)\,dt = \int_{40}^{50} .027 + .00025(t-40)^2\,dt$

(8) $= 10 \cdot .027 + .00025 \left.\frac{(t-40)^3}{3}\right|_{t=40}^{50} = .27 + .00025 \frac{(10)^3}{3}$

$= .27 + \frac{.25}{3} = 0.3533$

$\exp(-.3533) = .702$

b. $\exp\left(-\int_{40}^{60} \lambda(t)\,dt\right)$ $\int_{40}^{60} \lambda(t)\,dt = 20 \cdot (.027) + .00025 \left.\frac{(t-40)^3}{3}\right|_{t=40}^{60}$

(8) $= .54 + \frac{.00025}{3} \cdot 8000 = 1.2067$

$\exp(-1.2067) = 0.2992$

Math 3338 Homework Solutions

Ch 5, p 212

39. X is exp with $\lambda=1$, $Y=\ln X$. Find the pdf for Y.

$F_Y(y) = P(Y \le y) = P(\ln X \le y) = P(X \le e^y) = 1 - \exp(-e^y)$

(8) Then $f_Y(y) = \frac{d}{dy}\left(1 - \exp(-e^y)\right) = -\exp(-e^y)\cdot(-e^y)$

$= \exp(-e^y)e^y$