

# Math 3338 Homework Solutions

p 271 # 2a 3 balls chosen from urn without replacement,

urn contains 5 white 8 red balls.  $X_i = 1$  if  $i^{\text{th}}$  ball is white, 0 otherwise. Find joint pmf for  $X_1 + X_2$ .

Solution: The pair  $(X_1, X_2)$  has 4 possible values:

$(0, 0), (0, 1), (1, 0), (1, 1)$

$$P((X_1, X_2) = (0, 0)) = \frac{\binom{8}{2}}{\binom{13}{2}} = \frac{8 \cdot 7}{13 \cdot 12} = \frac{2 \cdot 7}{3 \cdot 3} = \frac{14}{39}$$

$$P((X_1, X_2) = (0, 1)) = P(X_1 = 0 \text{ and } X_2 = 1)$$

$$= P(X_2 = 1 | X_1 = 0) P(X_1 = 0) = \left(\frac{5}{12}\right) \cdot \left(\frac{8}{13}\right) = \frac{10}{39}$$

$$P((X_1, X_2) = (1, 0)) = P(X_1 = 1 \text{ and } X_2 = 0)$$

$$= P(X_2 = 0 | X_1 = 1) \cdot P(X_1 = 1)$$

$$= \left(\frac{8}{12}\right) \cdot \frac{5}{13} = \frac{10}{39}$$

$$P((X_1, X_2) = (1, 1)) = P(X_1 = 1 \text{ and } X_2 = 1)$$

$$= P(X_2 = 1 | X_1 = 1) = \left(\frac{4}{12}\right) \cdot \left(\frac{5}{13}\right) = \frac{5}{39}$$