Math 4377
February 15, 2019 Homework 5 Name ______________________________

Let $\beta = \left\{1,\ x,\ x^2\right\}$ be the standard basis of $P_2(\mathbb{R})$.

1. Let $\mathbf{T} : P_2(\mathbb{R}) \to P_2(\mathbb{R})$ be the linear transformation $\mathbf{T}(p(x)) = xp'(x)$. Let $\mathbf{S} : P_2(\mathbb{R}) \to P_2(\mathbb{R})$ be the linear transformation $\mathbf{S}(p(x)) = \frac{d}{dx}\left(xp(x)\right)$.
   (a) Find $[\mathbf{T}]_\beta$ and $[\mathbf{S}]_\beta$
   (b) Is $\mathbf{T}$ one-to-one?
   (c) is $\mathbf{T}$ onto?
   (d) is $\mathbf{S}$ one-to-one?
   (e) is $\mathbf{S}$ onto?

2. Let $\mathbf{P} : P_2(\mathbb{R}) \to P_2(\mathbb{R})$ project $P_2(\mathbb{R})$ on $\mathit{Span}\left\{1,\ x^2\right\}$ along $\mathit{Span}\left\{1-x\right\}$. Find $[\mathbf{P}]_\beta$. *Hint:* $\mathbf{P}(1)$ must be 1, What is $\mathbf{P}(x)$?

3. Let V be a vector space with finite dimension, and let $\mathbf{T} : V \to V$ be a linear transformation. Show that if $\mathbf{N}(\mathbf{T}) = N(\mathbf{T}^2)$, then $\mathbf{R}(\mathbf{T}) \cap \mathbf{N}(\mathbf{T}) = \{\mathbf{0}\}$. Show that as a consequence, $V = \mathbf{R}(T) \bigoplus \mathbf{N}(T)$.