Math 4377 Homework Solutions

HW #6

1a $\beta = \{(1,1),(-1,1)\}$ $\quad \beta' = \{(2,1),(1,2)\}$

Find change of coords matrix to change β' coordinates to β coordinates.

Solution: If $[v]_{\beta'} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$ then $v = c_1\begin{bmatrix} 2 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$

If $[v]_{\beta} = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$ then $v = d_1\begin{bmatrix} 1 \\ 1 \end{bmatrix} + d_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$

Then $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$, or

$$\begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = [v]_\beta = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\underbrace{\begin{bmatrix} c_1 \\ c_2 \end{bmatrix}}_{[v]_{\beta'}}$$

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So the change of coordinate matrix is

$$[I_{\mathbb{R}^2}]_{\beta'}^{\beta} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 3 & 3 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 3/2 & 3/2 \\ -1/2 & 1/2 \end{bmatrix}$$

b: $\beta = \{(1,0),(0,1)\}$

8 Solution: Now $[v]_\beta = v$ so $[v]_{\beta'} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}^{-1} v$, the change of coordinates matrix is

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}.$$

2. $T\begin{pmatrix} a \\ b \end{pmatrix} = \begin{bmatrix} 3a+b \\ -a+2b \end{bmatrix}$; $[T]_\beta = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ $\quad \beta$ = std basis for $\mathbb{R}^2$

$\beta' = \left\{\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix}\right\}$

Use Theorem 2.23 and $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$, find $[T]_{\beta'}$

Solution: By Theorem 2.23,

10 $\quad [T]_{\beta'} = Q^{-1}[T]_\beta Q. \quad [T]_\beta = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$Q = [I_{\mathbb{R}^2}]_{\beta}^{\beta'} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$

Then $[T]_{\beta'} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 4 & -2 \\ 1 & 3 \end{bmatrix}$

$$= \frac{1}{2}\begin{bmatrix} 5 & 1 \\ -3 & 5 \end{bmatrix} = \begin{bmatrix} 5/2 & 1/2 \\ -3/2 & 5/2 \end{bmatrix}$$