Math 4377
March 8, 2019 Homework 7 Name ______________________

1. Let $V = \mathbb{R}^2$ and let $\beta = \left\{ \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$. Find explicit formulas for vectors in the dual basis β^* for V^*, as in Example 4 p. 120.

2. Define $f \in \mathbb{R}^{2*}$ by $f(x, y) = x - 3y$ and $T : \mathbb{R}^2 \to \mathbb{R}^2$ by $T(x, y) = (x - 2y, 3y)$.
   (a) Compute $T^t(f)$.

   (b) Compute $[T^t]_{\beta^*}$, where β is the standard basis for $\mathbb{R}^2$ and $\beta^* = \{f_1,\ f_2\}$ is the dual basis, by finding scalars $a,\ b,\ c$ and d such that $T^t\,(f_1) = af_1 + bf_2$ and $T^t\,(f_2) = cf_1 + df_2$.

   (c) Compute $[T]_\beta$ and $\left([T]_\beta\right)^t$ and compare your results with (b).

3. Let

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 5 & 2 \end{bmatrix}.$$

Use elementary row operations to reduce A to I_3. Then use the elementary row operations to express A^{-1} as a product of elementary matrices. Then express A as a product of elementary matrices.