Math 4377 Homework Solutions

Hw #9

1. $R = \begin{bmatrix} 1 & -3 & 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & 0 & 4 & 5 \\ 0 & 0 & 0 & 1 & 6 & 7 \end{bmatrix}$

(4) a. Identify pivot columns and free columns and the corresponding pivot variable and free variables.

Answer: pivot cols are $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ #1, $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ #3, $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ #4

Free cols are $\begin{bmatrix} -3 \\ 0 \\ 0 \end{bmatrix}$ #2, $\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$ #5, $\begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}$ #6

Pivot variables are x_1, x_3, x_4

Free variables are x_2, x_5, x_6.

(9) b. Find the special solutions to $RX=0$

1. Let $x_2=1$, $x_5=x_6=0$. Then $x_1-3=0$, $x_1=3$, $x_3=x_4=0$

$$S_1 = \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Let $x_5=1$, $x_2=x_6=0$. Then $x_1+2=0$ $x_1=-2$ $x_4+6=0$ $x_4=-6$

$x_3+4=0$ $x_3=-4$

$$S_2 = \begin{bmatrix} -2 \\ 0 \\ -4 \\ -6 \\ 1 \\ 0 \end{bmatrix}$$

Let $x_6=1$, $x_2=x_5=0$. Then $x_1+1=0$ $x_1=-1$

$x_3+5=0$, $x_3=-5$. $x_4+7=0$, $x_4=-7$.

$$S_3 = \begin{bmatrix} -1 \\ 0 \\ -5 \\ -7 \\ 0 \\ 1 \end{bmatrix}.$$

(10) c. Show that $\{S_1, S_2, S_3\}$ is linearly dependent and is a basis for W?

Solution: If $c_1S_1+c_2S_2+c_3S_3=0$, then the second component of this equation is $c_1\cdot 1 + c_2\cdot 0 + c_3\cdot 0 = 0$ so $c_1=0$

The fifth component is $c_1\cdot 0 + c_2\cdot 1 + c_3\cdot 0 = c_2 = 0$

The sixth component is $c_3\cdot 1 = 0$ $c_3=0$

Thus $c_1S_1+c_2S_2+c_3S_3=0 \Rightarrow c_1=c_2=c_3=0$ - so $\{S_1, S_2, S_3\}$ is linearly dependent

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Hw #4.

1c. Continued Since B is a linearly independent set of 3 vectors in a vector space of dimension 3, B is a basis for $\mathbb{R}^3$.
↳ $\mathbb{R}^3$.

2. 3 Solutions to $Rx=0$, R in reduced echelon form

$$S_1 = \begin{bmatrix} -7 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad S_2 = \begin{bmatrix} -2 \\ 0 \\ -3 \\ 1 \\ 0 \end{bmatrix}, \quad S_3 = \begin{bmatrix} 6 \\ 0 \\ 5 \\ 4 \\ 1 \end{bmatrix}.$$

a. How many columns does R have? What is the rank of R?

Solution. Since $\{S_1, S_2, S_3\} \subset \mathbb{R}^6$, R has 6 columns.

(3) Since $\text{Null}(R)$ has dimension 3, $\text{rank } R = 6-3 = 3$.

b. Identify the pivot cols and free columns of R

Solution: Pivot columns $\#1, 3, 5$

(3) Free cols $\#2, 4, 6$

c. Find R

Solution Let $R = [r_1 \cdots r_6]$ r_1, r_3, r_5 are pivot columns of a matrix R in reduced echelon form, so

(12) $r_1 = e_1, r_3 = e_2, r_5 = e_3$.

Then $RS_1 = -7e_1 + r_2 = 0$ so $r_2 = 7e_1 = \begin{bmatrix} 7 \\ 0 \\ 0 \end{bmatrix}$

$RS_2 = -2e_1 - 3e_2 + r_4$. So $r_4 = 2e_1 + 3e_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$

$RS_3 = 6e_1 - 5e_2 - 4e_3 + r_6$ So $r_6 = -6e_1 + 5e_2 + 4e_3 = \begin{bmatrix} -6 \\ 5 \\ 4 \end{bmatrix}$

So $R = \begin{bmatrix} 1 & 7 & 0 & 2 & 0 & -6 \\ 0 & 0 & 1 & 3 & 0 & 5 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{bmatrix}$ (Assumes R has 3 rows)