Math 4377
April 15, 2019 Homework 9 Name ______________________

1. Let
$$A=\begin{bmatrix}4&0&0\\1&2&4\\0&0&4\end{bmatrix}=\begin{bmatrix}2&-4&0\\1&0&1\\0&1&0\end{bmatrix}\begin{bmatrix}4&0&0\\0&4&0\\0&0&-1\end{bmatrix}\begin{bmatrix}\frac{1}{2}&0&2\\0&0&1\\-\frac{1}{2}&1&-2\end{bmatrix}=PDP^{-1}.$$
Use the diagonalization to find the eigenvalues of A and a basis for each eigenspace.

2. Diagonalize $A=\begin{bmatrix}-4&-1\\1&-2\end{bmatrix}$ or explain why it is not diagonalizable.

3. Diagonalize $A=\begin{bmatrix}-4&2\\1&-2\end{bmatrix}$ or explain why it is not diagonalizable.

4. Let $T:P_3(\mathbb{R})\to P_3(\mathbb{R})$ by $T(f(x))=f(-x)$.
   (a) Find all eigenvalues of T.
   (b) Find all eigenvectors of T.
   (c) Find an ordered basis β for $P_3(\mathbb{R})$ such that $\mathbf{r}T_\beta$ is diagonal.

5. Prove that if an $n\times n$ matrix A is invertible and diagonalizable, then A and A^{-1} are simultaneously diagonalizable (see problems 17-19).