MATH 4377 EXAM REVIEW NAME________________

1. Label the following statements as true (T) or false (F). 2 pts each

T F a. It is impossible for a linear map T : $R^n \rightarrow R^n$ that the kernel N(T) and the range R(T) are the same subspace.

For the following, V and W are vector spaces with finite dimension, T: V $\rightarrow$ W is linear and β is a subset of V.

T F b. If β is linearly dependent then T(β) is linearly dependent.

T F c. If T(β) is linearly dependent, then β is linearly dependent.

T F e. If V and W have the same finite dimension and T: V → W is linear and onto, then T is an isomorphism.

T F f. If T and U are linear transformations from V to W and both agree on a linearly independent subset of V, then T = U.

T F g. If β spans V, T and U are linear transformations from V to W and both agree on β, then T = U.

For j-l, V, W, and Z are vector spaces with finite ordered bases α, β, and γ, respectively, T: V $\rightarrow$ W and U: W $\rightarrow$ Z are linear.

T F h. $\left[UT\right]_{\alpha}^{\gamma} = \left[T\right]_{\alpha}^{\beta}\left[U\right]_{\beta}^{\gamma}$

T F i. $\left[T(v)\right]_{\beta} = \left[T\right]_{\alpha}^{\beta}\left[v\right]_{\alpha}$

T F j. Assume V = W but α and β are different. If Q is the change of coordinate matrix that changes α coordinates into β coordinates, then $\left[T\right]_{\beta} = Q^{-1}\left[T\right]_{\alpha}Q$

2. Find the change of coordinates matrix that changes β coordinates into α coordinates, where β and α are the following bases of P_2.

$\beta = \left\{1,\ x,\ x^2\right\}$, $\alpha = \left\{1,\ 1+x,\ 1+x+x^2\right\}$. 20 pts

3. Determine whether the following linear transformations are invertible. Justify your answer.

a. $T: M_{2x2}(\mathbf{R}) \rightarrow M_{2x2}(\mathbf{R})$ defined by $T\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a+b & a \\ c & c+d \end{pmatrix}$ 15 pts

b. $T: M_{2x2}(\mathbf{R}) \rightarrow P_2(R)$ by $T\begin{pmatrix} a & b \\ c & d \end{pmatrix} = a + 2bx + (c+d)x^2$. 15 pts

4. Let $A=\begin{pmatrix} 1 & 3 \\ 1 & 1 \end{pmatrix}$ and $\beta=\left\{\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \end{pmatrix}\right\}$. Find $\left[L_A\right]_\beta$. Also find an invertible matrix Q such that $\left[L_A\right]_\beta = Q^{-1}AQ$. 20 pts

5. Let $T: P_2 \rightarrow \mathbf{R}$ by $T\left(p(x)\right) = \int_0^1 3p(x)\,dx$. Let $\beta = \{1, x, x^2\}$ be the standard basis of P_2 and $\alpha = \{1\}$ be the standard basis of R. Find $\left[T\right]_\beta^\alpha$. 10 pts