

Homework exercises on reduced echelon form

1. Let \mathbf{R} be the following matrix in reduced echelon form:

$$\mathbf{R} = \begin{pmatrix} 1 & -3 & 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & 0 & 4 & 5 \\ 0 & 0 & 0 & 1 & 6 & 7 \end{pmatrix}$$

- (a) Identify the pivot columns and free columns, and corresponding pivot variables and free variables.
- (b) Find the special solutions to $\mathbf{R}\mathbf{X} = \mathbf{0}$ corresponding to each free variable. One free variable is 1 and the other free variables are 0. Solve for the pivot variables.
- (c) Show that the set of these special solutions is linearly independent and is a basis for $N(\mathbf{R})$.
2. You are in contact with a secret agent in Ukraine. He sent you a list of special solutions to $\mathbf{R}\mathbf{X} = \mathbf{0}$, for a matrix \mathbf{R} in reduced echelon form, but he did not send you \mathbf{R} :

$$\mathbf{s}_1 = \begin{pmatrix} -7 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{s}_2 = \begin{pmatrix} -2 \\ 0 \\ -3 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{s}_3 = \begin{pmatrix} 6 \\ 0 \\ -5 \\ 0 \\ -4 \\ 1 \end{pmatrix}$$

- (a) How many columns does \mathbf{R} have? What is the rank of \mathbf{R} ?
- (b) Identify the pivot columns and free columns of \mathbf{R} .
- (c) Find \mathbf{R} .